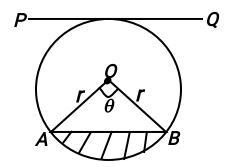
PARTS OF A CIRCLE

Summary:

1. The following are the main parts of a circle with centre O



(i) AB is called an arc

- (ii) OAB is called a sector
- (iii) line AB is called a chord
- (iv) line PQ is called a tangent
- (v) The shaded part is called a segment (vi) θ is the angle subtended by an arc

 - (vii) OA = OB = radius of the circle
- 2. Theorem: Since OA = OB, then OAB is an isosceles triangle with its line of symmetry bisecting chord AB
- 3. The following formulas are used in relation to the above circle:
 - (i) Circumference = $2\pi r$ or πd (ii) Area = πr^2
 - (iii) Arc length $AB = \frac{\theta}{360} \times 2\pi r$
 - (iv) Sector area **OAB**

$$=\frac{\theta}{360}\times\pi r^2$$

(v) Segment area = sector area - triangle area

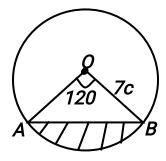
EXAMPLES:

1. Find the length of an arc which subtends an angle of 126° at the centre

1

of a circle of radius 14cm. Find also the length of the major arc

- 2. A chord is 8cm away from the centre of a circle of radius 17cm. Find the:
 - (i) length of the chord
 - (ii) size of the angle subtended by the chord at the centre
- 3. A chord of length 10cm is 12cm away from the centre of the circle. Find the:
 - (i) radius of the circle
 - (ii) length of the minor arc
 - (iii) area of the minor sector
 - (iv) area of the minor segment
- **4.** A sector of a circle of radius **12cm** has an angle of **150**° at the centre. The sector is folded to form a cone. Find the radius of the circular end of the cone
- 5. Find the distance between two parallel chords of lengths 32cm and 24cm which lie on opposite sides of the centre in a circle of radius 20cm
- 6. in the circle below, sector OAB has a radius of 7cm and subtends an angle of 120° at its centre 0.



Find the:

(i) shortest distance of chord AB from the centre

- (ii) perimeter of the shaded segment
- (iii) perimeter of the region enclosed between chord AB and the major arc
 - (iv) area of the shaded segment
- 7. The length of the common chord of two intersecting circles of radius 10cm and 17cm is 6cm. Find the:
- (i) angle subtended by the chord at the centre of the two circles
- (ii) area common to the two circles
- 8. Two equal circles of radius 5cm intersect at right angles. Find the:
- (i) distance between the centres of the two circles
- (ii) area common to the two circles
- **9.** A sector of a circle of radius **25cm** has an angle of **100·8°** at the centre. The sector is folded to form a cone. Find the:
- (i) radius of the circular end of the cone
- (ii) height of the cone
- (iii) volume of the cone
- (iv) total surface area of the cone
- HINT: (iv) $T \cdot S \cdot A = \pi r l$ since its circular end is open
- 10. The minor segment of a circle has a height of 4cm and a chord of length 16cm. Find the:
- (i) radius of the circle
- (ii) area of the segment

11. A dog tied on a rope 5m long is tethered to a tree 3m from a straight path. For what distance along the path is one in danger of being bitten by the dog?

EER:

- 1. A chord of length **70cm** subtends an angle of **120**° at the centre of the circle. Find the:
 - (i) radius of the circle
 - (ii) distance of the chord from the centre
 - (iii) area of the minor segment
- **2.** A chord of length **6cm** is **4cm** away from the centre of the circle. Find the circumference of the circle
- 3. A dog tied on a rope 2.5m long is tethered to a tree 2m from a straight path. For what distance along the path is one in danger of being bitten by the dog?
- **4.**A sector of a circle of radius **10cm** has an angle of **100**° at the centre. Find the
 - (i) perimeter of the sector
 - (ii) area of the minor segment
- **5.** A chord of length **6cm** makes an angle of **40**° with the radius of the circle. Find the circumference of the circle
- 6. A chord 3.5cm away from the centre of the circle subtends an angle of 120° at its centre. Find the area of the major segment
- 7. The chord of a circle of radius 10cm subtends an angle of 120° at its

centre. Find the perimeter of the region enclosed between the chord and the major arc

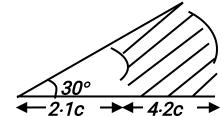
8. Find the distance between two parallel chords of lengths 32cm and 24cm which lie on the same side of the centre in a circle of radius 20cm

9. A sector of a circle of radius **12.5cm** has an angle of **100.8°** at the centre. The sector is folded to form a cone. Find the:

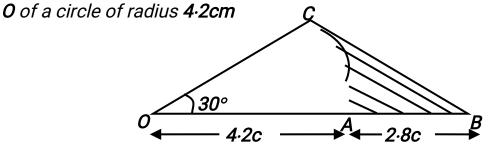
- (i) radius of the circular end of the cone
- (ii) height of the cone
- (iii) volume of the cone
- (iv) total surface area of the cone

10. Find the distance between two parallel chords of lengths 24cm and10cm which lie on opposite sides of the centre in a circle of radius 13cm

11. In the figure below, find the shaded area bounded by two concentric arcs

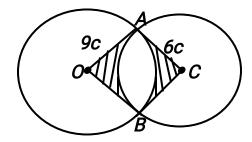


12. In the triangle OBC below, arc AC subtends an angle of 30° at the centre



If AB = 2.8cm, find the area of the shaded region

- 13. The length of the common chord of two intersecting circles of radius 28cm and 20cm is 30cm. Find the:
- (i) angle subtended by the chord at the centre of the two circles
- (ii) area common to the two circles
- **14.** The distance between the centres of two intersecting equal circles of radius **5cm** is **8cm**. Find the:
- (i) length of the common chord of the two circles
- (ii) area common to the two circles
- **15.** Two circles with centres **O** and **C** and radius **9cm** and **6cm** intersect at points **A** and **B** as shown

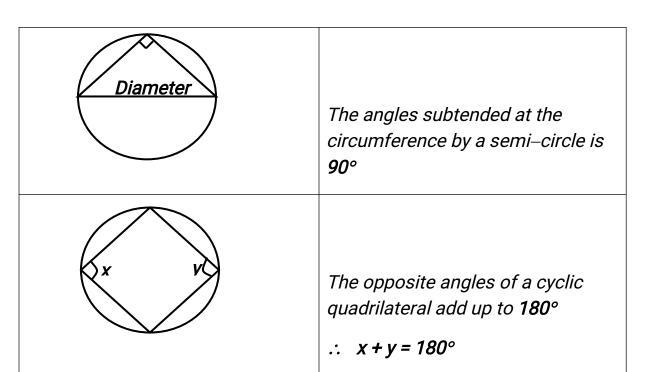


Given that the distance between O and C is 13cm, find the:

- (i) reflex angle AOB
- (ii) length of chord AB
- (iii) area of the shaded region

ANGLES IN A CIRCLE

Angles diagram	Circle theorems
A B	The angles subtended at the circumference by the same arc are equal
	∴ ∠APB =∠AQB
	In short:
	Angles in the same segment are equal
$\frac{P}{\theta}$ \frac{P}	The angle subtended at the centre by an arc is twice the angle it subtends at its circumference ∴ ∠AOB = 2∠APB
$A \longrightarrow B$	In short:
	Angle at the centre is twice the angle at the circumference

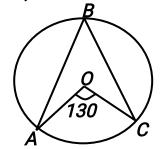


EXAMPLES:

1. In the figure below, 0 is the centre of the circle and $\angle AOC = 130^{\circ}$. Find

8

∠ABC

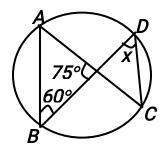


Soln:

$$\angle AOC = 2\angle ABC$$

$$\Rightarrow$$
 130° = 2 \angle ABC

2. In the circle below, find the size of angle marked x



Soln:

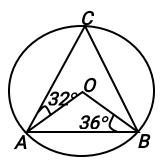
 $\angle BAC = x$ (angles in the same segment)

 \Rightarrow x = 180° - (60° + 75°) (angles in a triangle)

 $\therefore x = 45^{\circ}$

3. In the circle below 0 is its centre $\angle ABO = 36^{\circ}$ and $\angle OAC = 32^{\circ}$. Find

∠ACB and ∠OBC



Soln:

VAOB is isosceles since OA = OB

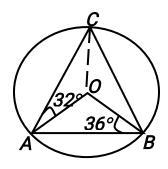
 \Rightarrow \angle **AOB** = 180° - (2 × 36°) = **108°** (angles in a triangle)

 $\angle AOB = 2\angle ACB$

 \Rightarrow 108° = 2 \angle ACB

∴ ∠ACB = 54°

Also:

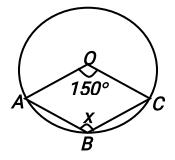


$$\Rightarrow$$
 \angle AOC = 180° - (2 × 32°) = **116°** (angles in a triangle)

$$\angle BOC = 360^{\circ} - (108^{\circ} + 116^{\circ}) = 136^{\circ}$$
 (angles at a point)

 \Rightarrow 2 \angle 0BC = 180 $^{\circ}$ – 136 $^{\circ}$ (angles in an triangle)

4. In the circle below **0** is its centre and $\angle AOC = 150^{\circ}$. Find the size of angle marked **x**



Soln:

Reflex angle $\angle AOC = 360^{\circ} - 150^{\circ} = 210^{\circ}$ (angles at a point)

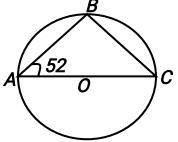
 $\Rightarrow \angle AOC = 2\angle ABC$

210° = 2∠ABC

∴ ∠ABC = 105°

5. In the figure below, **0** is the centre of the circle of radius **7cm** and $\angle BAC$

= 52°.



Find the:

- (i) size of angle ACB
- (ii) lengths of AB and BC

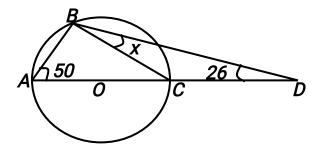
Soln:

:.
$$\angle ACB = 180^{\circ} - (90^{\circ} + 52^{\circ}) = 38^{\circ}$$
 (angles in a triangle)

(ii)
$$\cos 52 = \frac{AB}{14}$$

Also
$$\sin 52 = \frac{BC}{14}$$

6. In the circle below **0** is its centre. Find the size of angle marked x

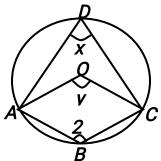


Soln:

$$\therefore x = 180^{\circ} - (90^{\circ} + 50^{\circ} + 26^{\circ}) = 14^{\circ}$$
 (angles in a triangle)

7. In the circle below **0** is its centre. Find the size of the angles marked ${\bf x}$

and y



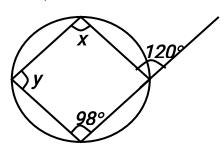
Soln:

$$x + 2x = 180^{\circ}$$
 (angles in a cyclic quadrilateral)

$$\therefore x = 60^{\circ}$$

Also
$$y = 2x = 2(60^{\circ}) = 120^{\circ}$$

8. In the circle below, find the size of the angles marked \mathbf{x} and \mathbf{y}



Soln:

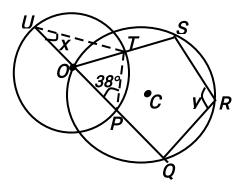
$$x + 98^{\circ} = 180^{\circ}$$
 (angles in a cyclic quadrilateral)

$$\therefore x = 82^{\circ}$$

Also
$$y + 60^{\circ} = 180^{\circ}$$
 (angles in a cyclic quadrilateral)

$$\therefore y = 120^{\circ}$$

9. In the diagram below, C and O are centres of two intersecting circles.



Find the size of the angles marked **x** and **y**

Soln:

(i) ∠UTP = 90° (angle in a semi–circle)

$$\therefore x = 180^{\circ} - (90^{\circ} + 38^{\circ}) = 52^{\circ}$$
 (angles in a triangle)

Also:
$$\angle POT = 180^{\circ} - (2 \times 38^{\circ}) = 104^{\circ}$$
 (angles in a triangle)

y + 104° = 180° (angles in a cyclic quadrilateral)

$$\therefore y = 76^{\circ}$$

10. The points P(-2, -1), Q(h, 7) and R(-3, 6) lie on a circle with diameter PQ.

- (i) State with a reason the size of angle PRQ
- (ii) Show that h = 4
- (iii) Find the coordinates of the centre and radius of the circle

Soln:

(i)
$$R(-3, 6)$$
 $Q(h, 7)$

∠PRQ = 90° (angle in a semi–circle)

(ii) Hint: This could be done using gradient method

Gradient of **PR** =
$$\frac{6 - 1}{-3 - 2} = -7$$

Gradient of **QR** =
$$\frac{7-6}{h-3} = \frac{1}{h+3}$$

For perpendicular lines, $-7 \times \left(\frac{1}{h+3}\right) = -1$

(iii) Hint: Centre is the midpoint of the diameter and radius is half the diameter

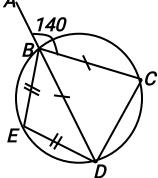
Centre =
$$\left(\frac{-2+4}{2}, \frac{-1+7}{2}\right) = (2, 3)$$

Radius =
$$\sqrt{(4-2)^2 + (7-3)^2}$$
 = 5 units

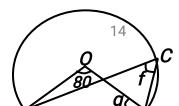
EER:

- 1. The vertices of an equilateral triangle A, B and C lie on a circle of radius 8cm. Find the:
- (i) distance of any side of the triangle from the centre of the circle
- (ii) length of the side of the triangle
- 2. The vertices of an equilateral triangle of side 12cm lie on a circle. Find the:
 - (i) radius of the circle
 - (ii) distance of any side of the triangle from the centre of the circle
 - (iii) area of the segments cut off by the triangle
- 3. In the circle below, BC = BD, BE= DE and ∠ABC = 140°. Find the size of





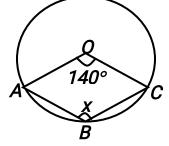
4. In the circle below **0** is its centre and $\angle AOB = 80^\circ$. Find the size of the



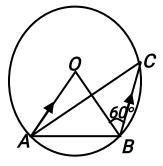
angles marked **f** and **g**

5. In the circle below 0 is its centre and $\angle AOC = 150^{\circ}$. Find the size of angle

marked **x**



6. In the circle below 0 is its centre, A0 is parallel to BC and $\angle OBC = 60^{\circ}$.



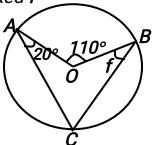
Find the size of angle:

(i) AOB

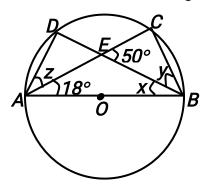
(ii) ACB

(iii) CAB

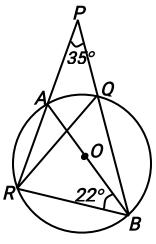
7. In the circle below 0 is its centre, $\angle AOB = 110^{\circ}$ and $\angle OAC = 20^{\circ}$. Find the size of angle marked f



8. In the figure below AB is the diameter of a circle centre O. $\angle BAC = 18^\circ$ and $\angle BEC = 50^\circ$. Find the size of the angles marked x, y and z



9. In the figure below **AB** is the diameter of a circle centre **0**. \angle **APQ** = **35**° and \angle **ABR** = **22**°.



Find the size of angle:

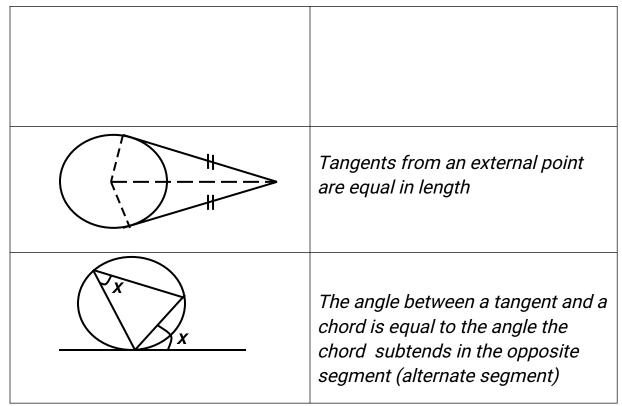
(i) ABQ

(ii) QRA

(iii) AOR

TANGENT PROPERTIES

Tangent diagrams	Circle theorems
	The angle between a tangent and the radius is 90 °

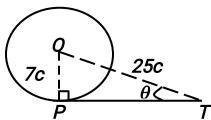


EXAMPLES:

- 1. A tangent from **T** to a circle, centre **O** and radius **7cm** touches the circle at **P**. If **OT = 25cm**, find the:
- (i) length of PT
- (ii) size of angle PTO
- (iii) area of PTO that lies outside the circle

Soln:

(i)



∠OPT = 90° (angle between a tangent and the radius)

:.
$$PT = \sqrt{25^2 - 7^2} = 24cm$$

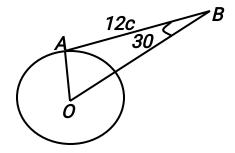
(ii)
$$\sin \theta = \frac{7}{25}$$

$$\theta = 16.26^{\circ}$$

(iii) POT =
$$180^{\circ} - (90^{\circ} + 16.26^{\circ}) = 73.74^{\circ}$$
 (angles in a triangle)

: Required area =
$$\frac{1}{2} \times 24 \times 7 - \frac{73 \cdot 74}{360} \times \frac{22}{7} \times 7^2 = 52 \cdot 4557 \text{cm}^2$$

2. In the circle below 0 is its centre. AB = 12cm is a tangent to the circle at $\angle OBA = 30^{\circ}$.



Find the:

- (i) length of OB
- (ii) radius of the circle

Soln:

(i) ∠OAB = 90° (angle between a tangent and the radius)

$$\Rightarrow cos30 = \frac{12}{OB}$$

(ii)
$$tan30 = \frac{OA}{12}$$

3. In the circle below 0 is its centre. AB and CB are tangents to the circle

and $\angle ABC = 56^{\circ}$. $D = 56^{\circ}$.

Find the size of angle ADC

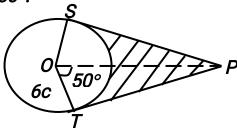
Soln:

$$\angle AOC + 90^{\circ} + 90^{\circ} + 56^{\circ} = 360^{\circ}$$
 (angles in a quadrilateral)

 \therefore \angle AOC = 124° (angles in a triangle)

If
$$\angle AOC = 2 \angle ADC$$

4. In the circle below **0** is its centre. **PT** and **PS** are tangents to the circle of radius **6cm** and $\angle POT = 50^{\circ}$.



Find the area of the shaded region

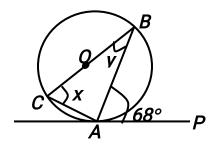
Soln:

(i) $\angle OTP = 90^{\circ}$ (angle between a tangent and the radius)

$$\Rightarrow tan50 = \frac{PT}{6}$$

: Shaded area =
$$2\left(\frac{1}{2} \times 7 \cdot 1505 \times 6\right) - \frac{100}{360} \times \frac{22}{7} \times 6^2 = 11 \cdot 4744cm^2$$

5. In the circle below 0 is its centre. AP is a tangent to the circle and $\angle PAB = 68^{\circ}$.



Find the size of the angles marked **x** and **y**

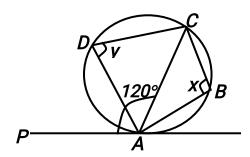
Soln:

x = 68° (angle in alternate segments)

∠BAC = 90° (angle in a semi–circle)

:
$$y = 180^{\circ} - (90^{\circ} + 68^{\circ}) = 22^{\circ}$$
 (angles in a triangle)

6. In the circle below AP is a tangent to the circle and $\angle PAC = 120^{\circ}$.



Find the size of the angles marked ${\bf x}$ and ${\bf y}$

Soln:

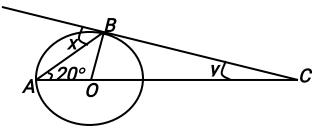
x = 120° (angle in alternate segments)

Also: $y + 120^{\circ} = 180^{\circ}$ (angles in a cyclic quadrilateral)

$$\therefore y = 60^{\circ}$$

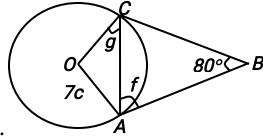
EER:

- 1. The tangents at **A** and **B** on a circle of radius 7cm intersect at **T**, and **C** is any point on the major arc AB. If \angle ATB = 48°, find the:
 - (i) size of angle ACB
 - (ii) area bounded by the tangents and the minor arc AB
- 2. In the circle below 0 is its centre. BC is a tangent to the circle and $\angle BAO = 20^{\circ}$.



Find the size of the angles marked x and y

- 3. The angles of a triangle are 50°, 60° and 70°, and a circle touches the sides at A, B and C. Find the angles of triangle ABC
- **4.** In the circle below **0** is its centre. **AB** and **CB** are tangents to the circle of radius 7cm and $\angle ABC = 80^{\circ}$.

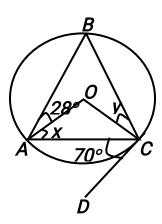


Find the:

- (i) size of the angles marked f and g
- (ii) size of the reflex angle AOC
- (iii) area bounded by the tangents and the minor arc AC

5. In the circle below O is its centre. DC is a tangent to the circle ∠BAO = 28°

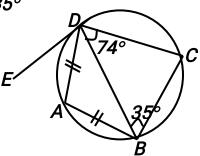
and ∠ACD = 70°



Find the size of the angles marked x and y

6. In the circle below ED is a tangent to the circle, AB = AD, ∠BDC = 74°

and ∠**DBC** = 35°



Find the size of angle:

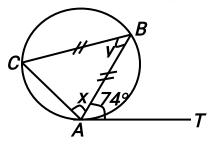
(i) DAB

(ii) BDE

(iii) DBA

(iv) EDA

7. In the circle below AT is a tangent to the circle, AB = CB and $\angle BAT = 74^\circ$. Find the size of the angles marked x and y

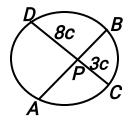


CIRCLES AND SIMILAR TRIANGLES

Circle diagrams	Analysis diagrams	Conclusion
"intersecting chords theorem"	C B B D	Triangles ACP and PBD are similar $\Rightarrow \frac{AP}{PD} = \frac{CP}{PB}$ $\therefore AP \cdot PB = CP \cdot PD$ "Each length is measured from the meeting point"
"intersecting secants theorem"	A B C	Triangles ACD and ECB are similar $\Rightarrow \frac{AC}{CE} = \frac{DC}{CB}$ $\therefore CA \cdot CB = CD \cdot CE$ "Each length is measured from the meeting point"
"intersecting tangent- secant theorem"	D C B	Triangles ABC and ABD are similar $\Rightarrow \frac{AB}{BC} = \frac{DB}{AB}$ $\therefore BA^2 = BD \cdot BC$ "Each length is measured from the meeting point"

EXAMPLES:

1. In the circle below chords AB and CD intersect at P. If CP = 3cm, DP = 8cm and AB = 10cm, find the length of AP



Soln:

If
$$AP = x$$
, $PB = 10 - x$

Using $AP \cdot PB = CP \cdot PD$ (intersecting chords theorem)

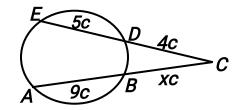
$$\Rightarrow x(10-x) = 3(8)$$

$$x^2 - 10x + 24 = 0$$

$$x = \frac{10 \pm \sqrt{100 - 96}}{2}$$

$$\therefore x = 6cm \text{ or } 4cm$$

2. In the circle below chords AB and ED are produced to intersect at C. If CD = 4cm, ED = 5cm, AB = 9cm and BC = xcm,



find the:

- (i) value of x
- (ii) ratio of the areas of triangle ACE to that of BCD
- (iii) area of ABDE if the area of triangle ACE is 54cm 2

Soln:

(i) Using $CB \cdot CA = CD \cdot CE$ (Each length from C)

$$\Rightarrow x(x+9) = 4(9)$$

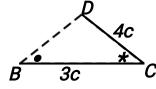
$$x^2 + 9x - 36 = 0$$

$$x = \frac{-9 \pm \sqrt{81 + 144}}{2}$$

$$x = 3$$
 or -12

$$\therefore x = 3$$

(ii) Using similar triangles ACE and BCD



$$\frac{Area \quad ACE}{Area \quad BCD} = \left(\frac{12}{4}\right)^2 = 9$$

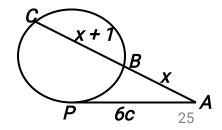
:: Required ratio = 9:1

(iii)
$$\frac{Area BCD}{54} = \left(\frac{4}{12}\right)^2$$

$$\therefore$$
 Area **BCD** = 6cm²

$$\Rightarrow$$
 Required area = 54 - 6 = 48cm²

3. In the circle below secant ABC intersects tangent AP at A. If AP= 6cm, AB = xcm and BC = (x + 1)cm, find the value of x



Soln:

Using $AB \cdot AC = AP^2$ (Each length from **A**)

$$\Rightarrow x(2x+1) = 6^2$$

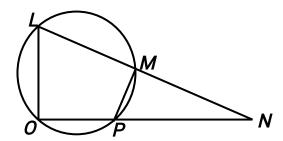
$$2x^2 + x - 36 = 0$$

$$x = \frac{-1 \pm \sqrt{1 + 288}}{4}$$

$$x = 4$$
 or -4.5

$$\therefore x = 4$$

4. In the circle below OL= 4.5cm, PM = 3cm, NM = 4cm and LN = 7.5cm.

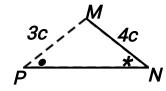


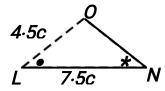
Find the:

- (i) lengths of ON and OP
- (ii) radius of the circle
- (iii) area of OLMP

Soln:

(i) Using similar triangles PMN and OLN,





$$\Rightarrow \frac{ON}{4} = \frac{4 \cdot 5}{3}$$

:. ON = 6cm

Also:
$$\frac{PN}{7 \cdot 5} = \frac{3}{4 \cdot 5}$$

:: PN = 5cm

$$\Rightarrow$$
 OP = ON - PN = 6 - 5 = 1cm

(ii) PMN is a right triangle based on its dimensions (∠PMN = 90°)

 \Rightarrow \angle POL = 90°, thus LP is a diameter (angle in a semi–circle)

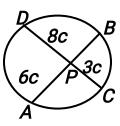
:
$$LP = \sqrt{4 \cdot 5^2 + 1^2} = 4 \cdot 6098cm$$

$$\Rightarrow$$
 Required radius = $\frac{1}{2} \times 4.6098 = 2.3049$ cm²

(iii) Required area =
$$\frac{1}{2} \times 6 \times 4 \cdot 5 - \frac{1}{2} \times 4 \times 3 = 7 \cdot 5 \text{cm}^2$$

EER:

1. In the circle below chords **AB** and **CD** intersect at **P**. If **CP = 3cm, DP = 8cm** and **AP = 6cm**,

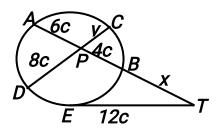


find the:

- (i) length of PB
- (ii) area of triangle BPC if the area of triangle APD is 20cm 2

(iii) ratio of the areas of triangle APC to that of BPD

2. In the circle below **TE = 12cm** is a tangent to the circle at **E**. Chords **AB** and **CD** intersect at **P** and **AB** is produced at **T**. If **AP = 6cm**, **PB = 4cm**, **BT = xcm**, **CP = ycm** and **DP = 8cm**,



find the:

(i) values of x and y

(ii) ratio of the areas of triangle APD to that of BPC

3. In the circle below **TE** is a tangent to the circle at **E**. Chords **AB** and **CD** are produced to intersect at **T**. If **AB** = **5cm**, **DC** = **9cm** and **CT** = **3cm**, find the lengths of **BT** and **ET**

